Enrollment No: ______________________ **Exam Seat No:** ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Group Theory
Subject Code: 5SC02GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 04/05/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:
(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) Define: Group **(01)**

b) Define: Index of subgroup of group. **(01)**

c) Define: Normalizer of element of group. **(01)**

d) If $(G,*)$ is a group and $a\,,b \in G$ then prove that $(a*b)^{-1} = b^{-1}*a^{-1}$ **(02)**

Let G be a group. If $(ab)^2 = a^2\,b^2\ \forall\ a,b \in G$ then prove that G is abelian. **(02)**

Q-2 Attempt all questions **(14)**

a) If $G = \left\{\begin{bmatrix} a & a \\ a & a \end{bmatrix} \,/\, a \text{ is non zero real number}\right\}$ then prove that G is group under multiplication. **(05)**

b) If $(G,*)$ be a group and $o(a) = n$ then prove the following **(05)**
(i) $o(a^q) \le o(a)$ for any integer q.
(ii) $o(a^{-1}) = o(a)$.

c) Define Centre of group G. Prove that Centre of group G is subgroup of G. **(04)**

OR

Q-2 Attempt all questions **(14)**

a) Prove that every cyclic group is an abelian group. Also prove that every subgroup of abelian group is normal. **(05)**

b) Prove that any two right cosets of a subgroup are either disjoint or identical. **(05)**

A homomorphism $\phi: (G,\cdot) \to (G',*)$ is one one if and only if $\ker\phi = \{e\}$. **(04)**

Q-3 Attempt all questions **(14)**

a) In usual notation prove that $I(G) \cong G|Z$. **(05)**

	b)	If a cyclic subgroup T of G is normal in G then prove that every subgroup of T is normal in G.	**(05)**
	c)	Define: Congruence relation. Prove that the congruent relation is an equivalence relation.	**(04)**

OR

Q-3	**Attempt all questions**		**(14)**
	a)	Define quaternion group. Prove that it is group under multiplication.	**(05)**
	b)	If in the group G, $a^5 = e, aba^{-1} = b^2$ for $a, b \in G$ find $o(b)$.	**(05)**
	c)	Prove that a group of prime order is cyclic.	**(04)**

SECTION – II

Q-4	**Answer the Following questions:**		**(07)**
	a)	Define: $p -$ sylow subgroup.	**(01)**
	b)	Define: Internal Direct product.	**(01)**
	c)	Write a class equation for group.	**(01)**
	d)	Let G be a group with order 12. Does there exists a subgroup of order 5? Justify your answer.	**(02)**
	e)	Let $o(G) = 120$. Does there exists a 5 – sylow subgroup of G? Justify your answer.	**(02)**

Q-5	**Attempt all questions**		**(14)**
	a)	State and prove fundamental theorem of homomorphism.	**(07)**
	b)	Let G be a group and suppose G is the internal direct product of $N_1, N_2, \dots, N_n$. Let $T = N_1 \times N_2 \times \dots \times N_n$. Then prove that G and T are isomorphic.	**(05)**
	c)	Let G be a group and ϕ is an automorphism of G. If $a \in G$ is of order $o(a) = n > 0$ then prove that $o\big(\phi(a)\big) = o(a)$.	**(02)**

OR

Q-5	**Attempt all questions**		**(14)**
	a)	Prove that each permutation $f \in S_n$ can be expressed as a composition of disjoint cycles.	**(07)**
	b)	Let G be group. For a fixed element g in G, define $\phi : G \to G$ by $\phi(x) = g\,xg^{-1}$. Prove that is ϕ an isomorphism of G on to G.	**(05)**
	c)	Find $o(f)$, where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 10 & 7 & 8 & 14 & 9 & 12 & 5 & 11 & 6 & 3 & 4 & 1 & 13 & 2 \end{pmatrix} \in S_{14}$	**(02)**

Q-6	**Attempt all questions**		**(14)**
	a)	State and prove Cayley's theorem.	**(07)**
	b)	List all conjugate classes in S_3, find c_a's for each class.	**(05)**
	c)	If order of group G is 49, then prove that G is abelian.	**(02)**

OR

Q-6	**Attempt all Questions**		**(14)**

a) State and prove Sylow's theorem **(07)**

b) If $o(G) = 24$ and G is abelian then how many subgroups of order 8? **(05)**

c) How many conjugate classes in S_4? **(02)**